Chapter 5
Properties of Triangles

Section 6
Indirect Proof and Inequalities in Two Triangles

GOAL 1: Using Indirect Proof

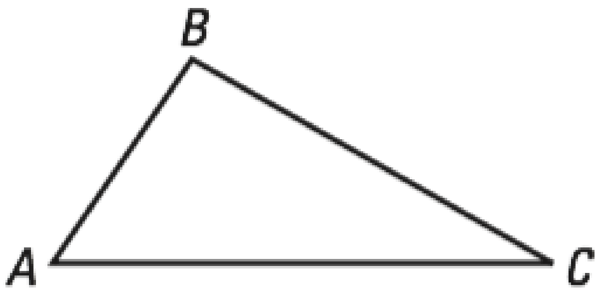
Up to now, all of the proofs in this textbook have used the Laws of Syllogism and Detachment to obtain conclusions directly. In this lesson, you will study indirect proofs.

An indirect proof is a proof in which you prove that a statement is true by first assuming that its opposite is true. If this assumption leads to an impossibility, then you have proved that the original statement is true.

Example 1: Using Indirect Proof

Use an indirect proof. Given: ΔABC

Prove: ΔABC does not have more than one obtuse angle.



Begin by assuming that $\triangle ABC$ *does* have more than one obtuse angle.

$m\angle A > 90°$ and $m\angle B > 90°$	**Assume $\triangle ABC$ has two obtuse angles.**
$m\angle A + m\angle B > 180°$	**Add the two given inequalities.**

You know, however, that the sum of the measures of all *three* angles is 180°.

$m\angle A + m\angle B + m\angle C = 180°$	**Triangle Sum Theorem**
$m\angle A + m\angle B = 180° - m\angle C$	**Subtraction property of equality**

So, you can substitute $180° - m\angle C$ for $m\angle A + m\angle B$ in $m\angle A + m\angle B > 180°$.

$180° - m\angle C > 180°$	**Substitution property of equality**
$0° > m\angle C$	**Simplify.**

The last statement is *not possible*; angle measures in triangles cannot be negative.

▶ So, you can conclude that the original assumption must be false. That is, $\triangle ABC$ cannot have more than one obtuse angle.

CONCEPT SUMMARY

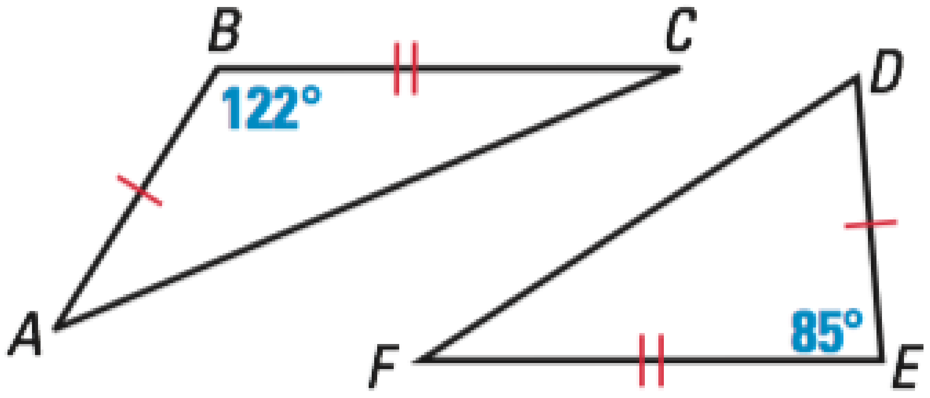
GUIDELINES FOR WRITING AN INDIRECT PROOF

1. Identify the statement that you want to prove is true.
2. Begin by assuming the statement is false; assume its opposite is true.
3. Obtain statements that logically follow from your assumption.
4. If you obtain a contradiction, then the original statement must be true.

GOAL 2: Using the Hinge Theorem

In the two triangles shown, notice that $\overline{AB} \cong \overline{DE}$ and $\overline{BC} \cong \overline{EF}$, but $m\angle B$ is greater than $m\angle E$.

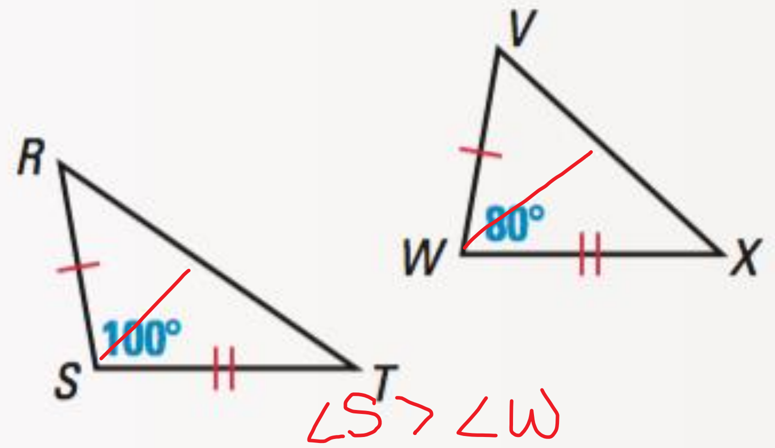
It appears that the side opposite the 122° angle is longer than the side opposite the 85° angle. This relationship is guaranteed by the Hinge Theorem below.



THEOREMS

THEOREM 5.14 ***Hinge Theorem***

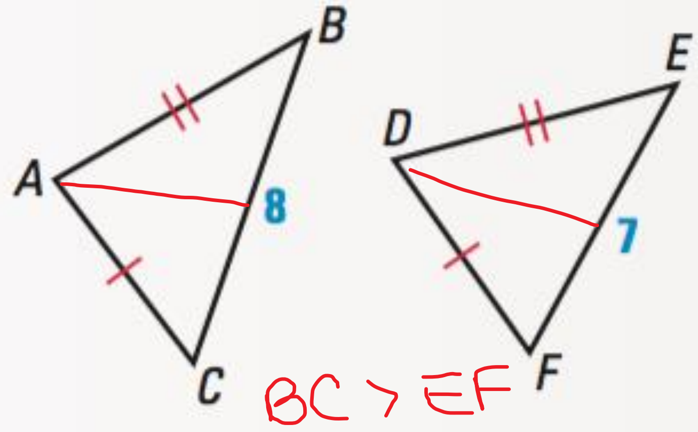
If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first is longer than the third side of the second.



$RT > VX$

THEOREM 5.15 ***Converse of the Hinge Theorem***

If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first is longer than the third side of the second, then the included angle of the first is larger than the included angle of the second.

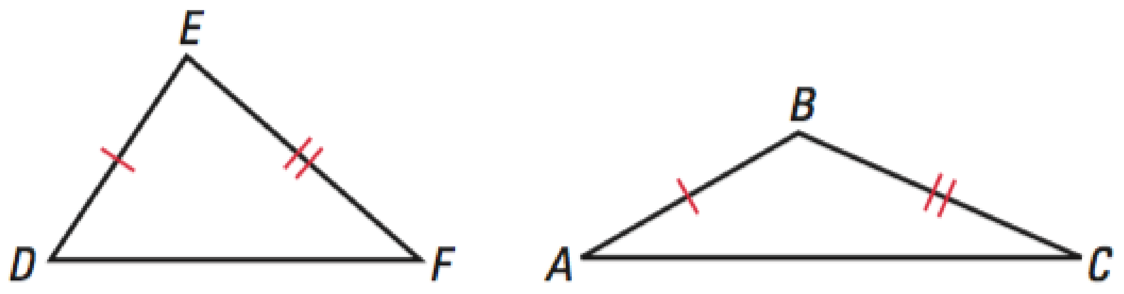


$m\angle A > m\angle D$

Example 2: Indirect Proof of Theorem 5.15

Given: $AB \cong DE$; $BC \cong EF$; $AC > DF$

Prove: $m\angle B > m\angle E$



SOLUTION Begin by assuming that $m\angle B \not> m\angle E$. Then, it follows that either $m\angle B = m\angle E$ or $m\angle B < m\angle E$.

Case 1 If $m\angle B = m\angle E$, then $\angle B \cong \angle E$. So, $\triangle ABC \cong \triangle DEF$ by the SAS Congruence Postulate and $AC = DF$.

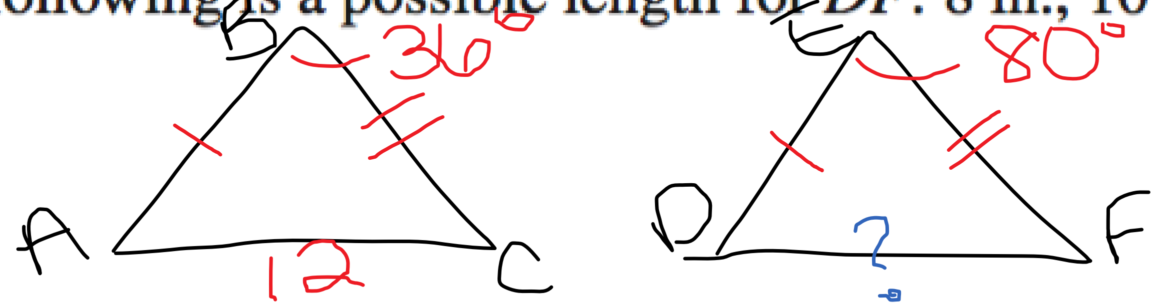
Case 2 If $m\angle B < m\angle E$, then $AC < DF$ by the Hinge Theorem.

Both conclusions contradict the given information that $AC > DF$. So the original assumption that $m\angle B \not> m\angle E$ cannot be correct. Therefore, $m\angle B > m\angle E$.

Example 3: Finding Possible Side Lengths and Angle Measures

You can use the Hinge Theorem and its converse to choose possible side lengths or angle measures from a given list.

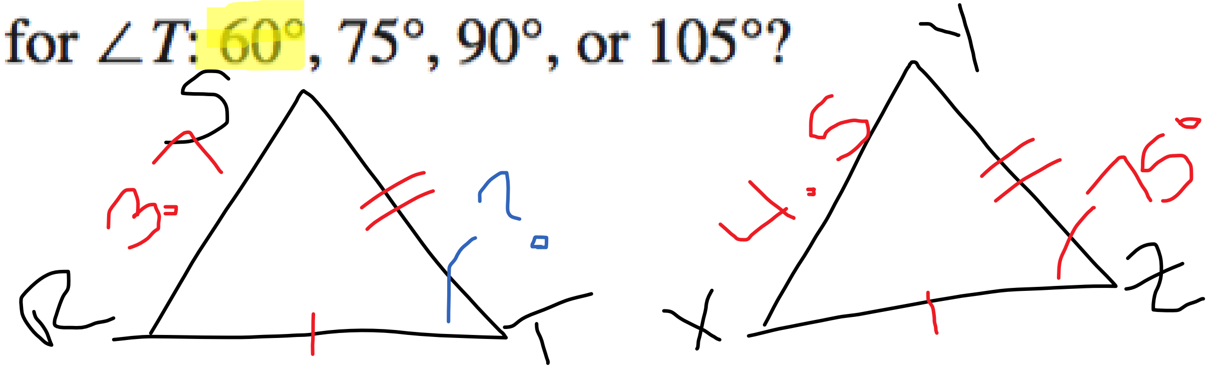
a. $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $AC = 12$ inches, $m\angle B = 36°$, and $m\angle E = 80°$. Which of the following is a possible length for $\overline{DF}$: 8 in., 10 in., 12 in., or 23 in.?



$\angle E > \angle B$

$DF > AC$

b. In a $\triangle RST$ and a $\triangle XYZ$, $\overline{RT} \cong \overline{XZ}$, $\overline{ST} \cong \overline{YZ}$, $RS = 3.7$ centimeters, $XY = 4.5$ centimeters, and $m\angle Z = 75°$. Which of the following is a possible measure for $\angle T$: 60°, 75°, 90°, or 105°?



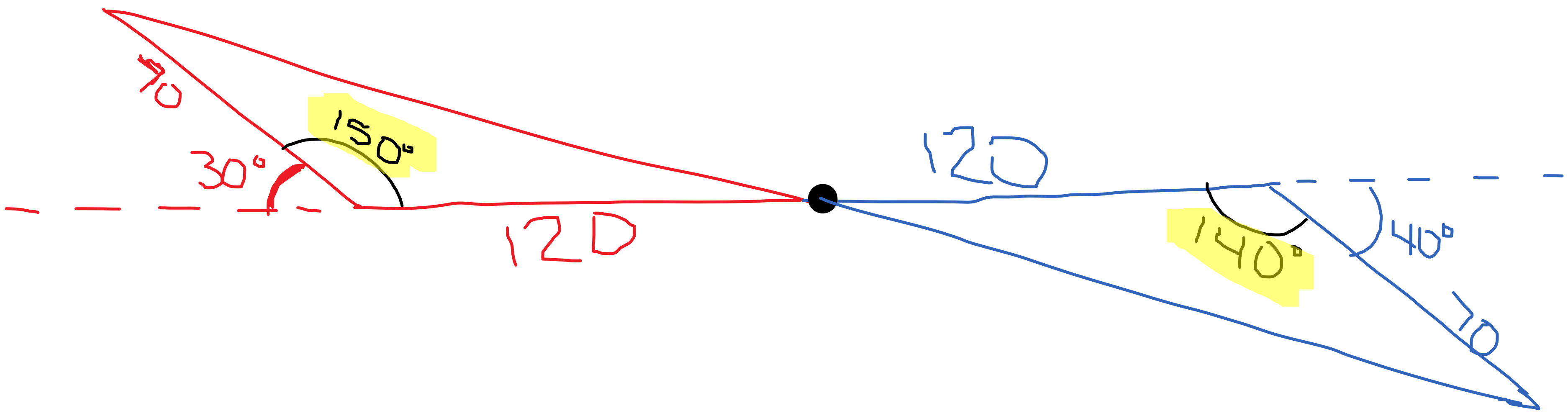
$XY > RS$

$\angle Z > \angle T$

Example 4: Comparing Distances

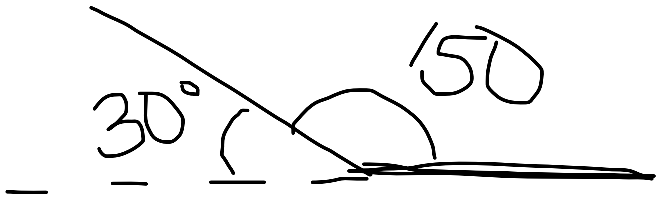
TRAVEL DISTANCES You and a friend are flying separate planes. You leave the airport and fly 120 miles due west. You then change direction and fly W 30° N for 70 miles. (W 30° N indicates a north-west direction that is 30° north of due west.) Your friend leaves the airport and flies 120 miles due east. She then changes direction and flies E 40° S for 70 miles. Each of you has flown 190 miles, but which plane is farther from the airport?

we are further



EXIT SLIP

W 30° N



N 30° W